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Generalized Production Function and Factor-Intensity Crossovers: An Empirical Analysis

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No. 30

I Introduction

In international trade theory, the theoretical possibility of factor reversals (or factor-intensity crossovers) is important as a possible explanation of the "Leontief paradox" in the empirical test of the Heckscher-Ohlin Theory [3] and of the non-existence of factor-price equalization. With the discovery of the CES or homohypallagic production function [1], it has been found that this production function, which possesses the traditional properties of linear homogeneity and positive and diminishing marginal productivities of factors, can give rise to the phenomenon of factor reversal. The CES production function has therefore been used as a basis for empirically testing the existence of factor reversals. Minhas [6 and 7] found, in a study involving international comparisons, that factor reversals are quite common within the empirically relevant range. His conclusion was, however, contested by Leontief [4] who found, by recalculating some of Minhas' results with appropriate modifications, that the possiblity of factor reversals is much less than Minhas suggested. The empirical question of the presence of factor reversals is thus unresolved. More recently, in a study using data for nine developed countries, and employing the CESh production function (a limited extension of the original CES production function to accommodate degrees of homogeneity other than unity--the case of non-constant returns to scale), Philpot [8] found that all but one of the industries examined by him have the same (or almost the same) elasticity of factor substitution, and therefore concluded that factor reversal does not occur.

In this study we will take another look at the factor reversal question by applying a more general production function--a VESh (or "hetero-hypallagic") production function consistent with homogeneity of degree h--in the empirical analysis. Section I will discuss briefly the generation of the generalized production function to be used. In Section II, the estimation problems will be dealt with. Only U. S. data which are more readily accessible are used. We will present and interpret our findings about the number of factor-intensity crossovers within the empirically relevant range in Section III. This will be followed by a summary and some concluding remarks.

II Derivation of the Generalized Production Function

Let the production function be of the general form $V = F(K,L)$. When the function F is homogeneous of degree h, we can write

$$(2\text{-}1) \qquad V = L^h F(K/L,1).$$

Putting $y = V/L^h$ and $x = K/L$, then

$$(2\text{-}2) \qquad y = f(x).$$

Under the assumption of perfectly competitive markets, the real wage rate is equal to the marginal product of labor, i. e.

$$(2\text{-}3) \qquad w = \partial V/\partial L = \partial(L^h y)/\partial L = L^{h-1}(hy - x\, dy/dx).$$

Thus,

$$(2\text{-}4) \qquad w/L^{h-1} = hy - x\, dy/dx.$$

Starting with an empirical relation of the following form,

$$\log V/L = \log a + b \log w + c \log K/L + u$$

Lu and Fletcher [5] derived the VES production function:

$$V = [\beta K^{-\rho} + \alpha\eta L^{-\rho} (K/L)^{-c(1+\rho)}]^{-1/\rho}$$

Suppose we specify instead

$$(2\text{-}5) \qquad \log V/L = \log a + b \log w + c \log K/L + d \log L + u$$

where the addition of the log L term may be justified by the test of the statistical significance of d. Furthermore, let $d = (h-1)(1-b)$.[1] The economic significance of this will become clear as we proceed. Equation (2-5) can now be expressed as

$$\log V/L^h = \log a + b \log w/L^{h-1} + c \log K/L + u.$$

---

[1] Lu and Fletcher's empirical relation $\log V/L = \log a + b \log w + c \log K/L + u$ may be equivalently written in autilogarithmic form (with the disturbance term left out for convenience) as $V/L = a\, w^b (K/L)^c$, or $V/L^h = a\, (w/L^{h-1})^b (K/L)^c L^{(1-h)(1-b)}$. In our specification of d we took the cue from the exponent of the L term.

If we use (2-4) for substitution and leave out the disturbance term for convenience,

$$(2\text{-}6) \qquad \log y = \log a + b \log (hy - x\, dy/dx) + c \log x.$$

Clearing the equation of logarithms, we get

$$y = a(hy - x\, dy/dx)^{b}\, x^{c}.$$

By a slight change of notation and rearranging terms, a differential equation of y(x) is obtained:

$$(2\text{-}7) \qquad dy/dx = y/x\, [h - \alpha x^{-v}\, y^{(1/b)-1}]$$

where we have set $\alpha = a^{-1/b}$ and $v = c/b$. Equation (2-7) is nonlinear and thus is difficult to solve directly. Yet it can be transformed into a linear differential equation by substituting $z = y^{1-(1/b)}$. Thus (2-7) becomes

$$(2\text{-}7a) \qquad dz/dx = [(1/b)-1]\, \frac{h}{x}\, z = [(1/b) - 1]\, \alpha x^{-(v+1)}.$$

Using the integrating factor $e^{\int [(1/b)-1]\, (h/x)\, dx}$ and carrying out the details, the solution is given by

$$z = x^{-h[(1/b)-1]} \{ \frac{1-b}{h(1-b)-c}\, \alpha x^{h[(1/b)-1]-v} + \beta \}$$

where β is the constant of integration. To economize on notation, set $\rho = h[(1/b)-1]$, and $\eta = \frac{1-b}{h(1-b)-c}$.

Then the solution becomes

$$z = x^{-\rho}\, [\alpha\eta x^{\rho-v} + \beta].$$

Returning to the original variables V, K, and L, we get

$$(2\text{-}8) \qquad V = [\beta K^{-\rho} + \alpha\eta L^{-\rho}\, (K/L)^{-v}]^{-h/\rho}$$

That (2-8) is homogeneous of degree h can be readily verified. When h = 1, it reduces to the familiar VES function. Therefore, we shall refer to the production function represented by (2-8) as the VESh pro-

duction function.

To show that the elasticity of substitution (σ) obtained from the VESh production function is variable, first find the marginal rate of substitution (s) as follows:

$$(2\text{-}9) \qquad s = -dK/dL = \frac{\partial V/\partial L}{\partial V/\partial K} = \frac{\alpha\eta(\rho-v)\ x^{\rho-v+1}}{\beta\rho + \alpha\eta v x^{\rho-v}}$$

The elasticity of substitution, a theoretical concept which measures the ease of factor substitution, is defined as the percentage change in the factor ratio in response to one per cent change in their marginal rate of substitution; thus

$$\sigma = (dx/ds)\ (s/x)$$

Given (2-9), it will be found that

$$(2\text{-}10) \qquad \sigma = \frac{1}{1 + \dfrac{\beta\rho(\rho - v)}{\beta\rho + \alpha\eta v x^{\rho-v}}}$$

Being a function of the capital-labor ratio, the elasticity of substitution is obviously not a constant.

The VESh production function is the general form of a class of homogeneous production functions. It is interesting to point out several special but familiar cases of the VESh production function:

(i) If $h = 1$, equation (2-8) is a linear homogeneous VES production function (now seen as an acronym of a VES1).

(ii) If $v = 0$, equation (2-8) reduces to the CESh production function. If furthermore $h = 1$, then the celebrated CES form results (now seen as an acronym of a CES1).

(iii) As to the special CES form, it is well known that it includes the Cobb-Douglas and fixed-coefficient (Walras-

Leontief-Harrod-Domar) production functions as limiting cases. (See [1]).

III Empirical Analysis and Estimation

Specification-error analysis

The derivation of the VESh production function presumes the statistical validity of the empirical relationship (2-5), i.e.

$$\log V/L = \log a + b \log w + c \log K/L + d \log L + u.$$

When the CES function was derived, the authors [1] assumed that the average productivity of labor depends only on the real wage rate:

$$\log V/L = \log a' + b' \log w + u' \tag{3-1}$$

At the same time, it was shown that the partial regression coefficient of log w (i.e., b') is equal to the elasticity of substitution. If assumption (3-1) does not hold, then b' may not represent the true elasticity of substitution. From the general viewpoint of specification-error analysis, the omission of log K/L and log L terms in the regression equation (2-5) will cause biasedness in the estimate of b', and hence the elasticity of substitution.

Two step estimation procedure

The VESh production function, which is reproduced here has six parameters--β, α, ρ, v, η, and h:

$$V = [\beta K^{-\rho} + \alpha\eta L^{-\rho}(K/L)^{-v}]^{-h/\rho}. \tag{3-2}$$

A salient feature that distinguishes (3-2) from the prevailing production functions is that nonlinearity exists in both the parameters and the variables. The ordinary least squares method is thus not applicable. Several nonlinear estimation procedures are now available for econometric analysis. In general, most nonlinear estimation techniques are essentially gradient methods which either maximize the likelihood function

or minimize the sum-of-squares expression. For simple functional forms, these procedures are quite efficient. Two disadvantages are noted in all gradient methods, however. First the speed of convergence depends heavily on the initial guess of the parameter values. If the surface generated by the function is highly irregular, convergence may come about only after a large number of iterations. A "bad" choice of initial values tends to lengthen the time for convergence. Most important of all, there is no guarantee that the solution, once convergence is attained, is the global optimum. One way to overcome this difficulty is to repeat the estimation from different sets of initial values; yet this is not always economically feasible for a study that involves a large number of regressions.

A simple two-step procedure is found appropriate for our purpose. Although there are six parameters to be estimated in the production function (2-8), we know, from the derivation in Section II, that h, ρ, η, and v are simple functions of b, c, and d in the empirical relation (2-5). Thus only α and β remain to be found.

First-step regressions--Equation (2-5) is a log-linear relationship. It is hence straightforward to obtain the least-squares estimates for b, c, and d. For a systematic study of the statistical justification of log K/L and log L, we performed regressions of three basic types:

(Type I) $\log V/L = \log a_1 + b_1 \log w + u_1$

(Type II) $\log V/L = \log a_2 + b_2 \log w + c_2 \log K/L + u_2$

(Type III) $\log V/L = \log a_3 + b_3 \log w + c_3 \log K/L + d_3 \log L + u_3$

Cross-section data for 1957 on 17 U. S. two-digit manufacturing industries by states [2] were used for the regression analyses. The data covered output, three different measures of capital, two measures of employment and their corresponding wage rates. (The meaning and explanation of these variables are given in the Appendix-A). By combining the different measures of capital and labor and the corresponding wage rate in different ways, 14 regressions were run (for details, see the Appendix-B). Significance test were performed for the estimated parameters, b_i, c_i, and d_i.[1] A summary of the respective numbers of statistically significant coefficients at the 5% and 10% levels is given in Table 3.1.

In Type I regressions approximately 12 out of 17 b_1 values are significantly different from zero at the 5% level. But the coefficient of determination is rather small. The addition of a capital variable (i.e. log K/L) in Type II regressions generally improves the goodness of fit. About 7 out of 17 c_2 values are significantly different from zero at the 5% level; this finding is in agreement with the result of Lu and Fletcher [5] Finally in Type III regressions, the significance of d_3 cannot be generally established due to the small number of significant values.[2]

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[1] Under the assumption that the error term has a log-normal distribution, the usual Students' t-test is appropriate. In so doing, the form of the production function may be tested. To give some examples, the null hypothesis that $d_3 = 0$ provides the test on whether returns to scale is constant; that $c_i = 0$ provides the test on whether the production function is of CES or VES form; that $b_i = 0$ when $c_i \neq 0$, or that $b_1 = 1$, provides the test for the Cobb Douglas form; and that $b_i = 0$ when $c_i = 0$ provides the test for the case of fixed input coefficients.

[2] Our tests are therefore not conclusive enough to reject the hypothesis that constant returns to scale is the prevailing mode of production in U. S. manufacturing industries.

Table 3.1*

Type	Reg. No.	Number of significant coefficients for the logarithms of																			
		w		w_p		K_g/L		K_g'/L		K_n/L		K_g/L_p		K_g'/L_p		K_n/L_p		L		L_p	
I	1	12	1																		
	2			12	1																
II	3	9	2			8	1														
	4	8	2					7	0												
	5	11	0							9	1										
	6			9	1							5	4								
	7			7	2									6	1						
	8			10	2											8	2				
III	9	7	1			7	1											2	1		
	10	7	3					7	0									1	2		
	11	8	1							8	1							1	1		
	12			9	1							5	2							1	1
	13			7	0									5	2					2	1
	14			9	2											6	4			1	4

The first column under each variable indicates the number of significant values at the 5% level; he second column indicates the number of additional ones at the 10% level. Please refer to the ppendix for the meaning of both the regressions and the variables.

It may also be noted that the number of significant b values tends to be overstated in Type I regressions in comparison with Type II, and also in Type II in comparison with Type III. This result arises from the problem of specification error.

The parameters b, c, and d having been obtained, the estimates for h, ρ, η, and v are then given as:

$$\begin{aligned} \hat{h} &= 1 + \hat{d}/(1-\hat{b}) \\ \hat{\rho} &= \hat{h}(1-\hat{b})/\hat{b} \\ \hat{\eta} &= (1-\hat{b})/[\hat{h}(1-\hat{b}) - \hat{c}] \text{ and} \\ \hat{v} &= \hat{c}/\hat{b} \end{aligned} \tag{3-3}$$

Second-step regressions--In the second step of our estimation procedure, consider the values of the four parameters in (3-3) as prior information and hold them fixed in the production function (3-2), which can thus be rearranged as:

$$V^{-\hat{\rho}/\hat{h}} = \beta K^{-\hat{\rho}} + \alpha\eta L^{-\hat{\rho}} (K/L)^{-\hat{v}}$$

By letting

$$V^* = V^{-\hat{\rho}/\hat{h}}, \quad K^* = K^{-\hat{\rho}}, \text{ and } L^* = \hat{\eta} L^{-\hat{\rho}} (K/L)^{-\hat{v}}$$

we get a simple linear regression equation of V^* and K^* and L^*:

$$V^* = \beta K^* + \alpha L^*. \tag{3-4}$$

β and α can now be estimated by applying ordinary least squares to equation (3-4). The estimation of the VESh production function is hence complete.

IV Factor-Intensity Crossovers

The unambiguous factor-intensity assumption plays an important role in many theorems of trade and development. In particular, the validity of the Heckscher-Ohlin theory of international trade and universal factor price equalization depends, in a crucial way, on the industries' comparative positions on the factor-intensity scale. Yet it is not always possible to characterize industries as "capital-intensive" or "labor-intensive" irrespective of the factor price ratio; the industries often adjust their factor-intensity ratios in response to changes in the relative factor prices. Subsequently it is possible that their existing factor intensities may be reversed. Minhas [6 and 7] demonstrated the factor reversal phenomenon with the CES function. He found that, for the CES, the factor-intensity ratio, (K/L), and the factor-price ratio, (w/r), maintain a log-linear relation for each industry. This relation between K/L and w/r may be represented by a straight line if we plot the function (loosely termed the "substitution function") on a double-logarithmic scale. Since two straight lines, unless parallel to each other by coincidence, are bound to intersect, the CES was used as a case against the "strong factor intensity assumption." In his review of Minhas' book [7], Leontief [4] modified the estimation procedure and arrived at a different conclusion. By plotting the "substitution functions" for 21 industries, only 17 out of 210 theoretically possible crossover points fell within the empirically relevant range of factor-price ratios, spanned on the one end by those observed in India and on the other by those in the United States. Moreover most of the cross-

over points within the relevant range belonged to curves which ran so closely together that for all practical purposes their capital-labor intensities would be considered identical. Hence the Heckscher-Ohlin theory of international trade appears to have been vindicated.

The Cobb-Douglas production function precludes the possibility of factor-intensity reversal since all "substitution functions" have equal slopes of unity and will never intersect. A distinctive feature of the CES is that it allows for interindustry differences in the relative ease or difficulty with which factor inputs can substitute for each other. The elasticity of substitution is nonetheless a constant for any factor-intensity ratio. Thus the factor-intensity ratio changes at a constant rate (σ) as the factor-price ratio changes.

Three weaknesses are noticed in the Minhas-Leontief analysis. First, due to the lack of international data on the capital variable, certain indirect procedures had to be used to estimate the intercept of the "substitution function" (see Leontief [4]). The use of indirect estimation procedures is likely to cause biasedness in the estimates. Second, their analysis assumes that all industries have the same (CES) form of the production function, which is unrealistic. Third, their estimates of the coefficient of log w may be biased owing to specification error as we have noted above.

In our analysis, the VESh production function is used such that the elasticity of substitution is free to vary at different factor-intenaity ratios. Data on capital for the 17 manufacturing industries are available in the U. S. census of 1957 and are used to estimate the

production function parameters directly. Furthermore, the industry production function is not restricted to a certain form because the VESh function may be reduced to fixed-coefficient, Cobb Douglas or CES form based on empirical results.

If a profit-maximizing industry considers the factor prices (w and r) as given, it will employ capital and labor in such amounts as to equate the price ratios, w/r, to the marginal rate of substitution, s. According to (2-9), the "substitution function" for the VESh production function is:

$$(4\text{-}1) \qquad w/r = \frac{\alpha\eta(\rho - v)\, x^{\rho - v + 1}}{\beta\rho + \alpha\eta v\, x^{\rho - v}}$$

If $v = 0$, equation (4-1) becomes

$$(4\text{-}2) \qquad w/r = (\alpha\eta/\beta)x^{1+\rho}$$

which is the "substitution function" for the CES case. If both $v = 0$ and $\rho = 0$, then the "substitution function" for the Cobb-Douglas case results:

$$(4\text{-}3) \qquad w/r = (\alpha\eta/\beta)x.$$

As to the limiting fixed-coefficient case, the "substitution function" is

$$(4\text{-}4) \qquad x = (\text{a constant}).$$

It should be seen that, of the four types of "substitution functions" shown above, only (4-1) gives a non-linear relationship between log x and log w/r.

Associated with each production function is a "substitution function" which assumes one of the forms given by equations (4-1) to (4-4).

In each of the 14 regressions indicated in Section III, the production function parameters were estimated for each of the 17 industries. Using these estimates, the "substitution functions" were formulated. Corresponding to each regression we plotted (by computer) the "substitution curves" for the 17 industries. The number of factor-intensity crossover points located within the "relevant range" for w/r (exactly the same range as that used in the Minhas-Leontief analyses) was then counted.

To facilitate comparison, two cases were distinguished. In Case 1, the tests of significance on the regression coefficients were ignored. Thus, for example, Type I regressions would always generate an assumed CES production function irrespective of the statistical significance of b. For each regression, this procedure would yield the same production function for all industries.

In Case 2, the statistical tests of significance at the 10% level were considered for all first-step regression coefficients when we determined the form of the production function for each industry. Thus, for example, in Type I regressions, an industry production function is considered as CES if b is significant, and to be of the fixed-coefficient form if b is not significant. This procedure allows for production functions to vary in form in different industries.[1] The approach

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[1] It may be noted from (3-3) that if $\hat{b} = 0$, then $\hat{\rho}$ becomes undefined. In the case of Type I regressions which corresponds to the CES form, we know that as b approaches 0, then the production function reduces to the fixed-coefficient case. The same result obtains in Types II and III regressions if b approaches 0 while c = 0. Under these conditions the K/L ratio is independent of w/r, and therefore the relevant "substitution function" is given by equation (4-4). The constant value of the K/L ratio is computed by taking the simple arithmetic average of the observed sample K/L values for the industry concerned.

In the case where b_i approaches zero while $c_i \neq 0$, equation (2-5) may be equivalently written as $V = a\,(K/L)^{c}L^{1+d}$. In this case it can be shown that the corresponding "substitution function" can be more conveniently computed by $w/r = \frac{1+d-c}{c}x$.

followed in Case 2 can therefore be considered more realistic compared with Case 1. Note that within Case 2, Type III regressions are more general than Type II, which in turn is more general than Type I.

Table 4.1 gives a brief summary of the number of factor-intensity crossovers for both cases. Several comments are in order. In all but three instances, the number of crossovers in Case 1 are smaller than those in Case 2.[1] This is an indication that a tendency may exist for Case 1 to understate the number of crossover points within the "relevant range."

Also from Case 1 (which maintains the same production function form for all industries studied under each of the three regression types), it can be observed that there are wide differences in the number of crossovers between Types II and III, although the difference is not evident between them and Type I. This limited evidence, nevertheless, suggests that the number of crossovers may be sensitive to the specification of the form of the production function.

Turning to Case 2, the number of crossover points does not appear to differ significantly between Types II and III[2]. This can be explained by recalling that the additional log L term in Type III regressions is seldom significant (see Table 3-1), so that empirically speaking Types II and III are not vastly different. As can therefore be expected, the

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[1]The similarity in the number of crossover points between Cases 1 and 2 is rejected by the "Sign Test" at the 5% significance level.

[2]The "Sign Test" was applied here, and was rejected at the 10% significance level.

Table 4.1*

Type	Regression number	Number of Factor-Intensity Crossovers Case 1	Case 2
I	1	71	67
	2	60	63
II	3	56	68
	4	64	77
	5	48	59
	6	55	60
	7	69	71
	8	53	57
III	9	73	69
	10	75	77
	11	70	72
	12	50	68
	13	61	53
	14	59	62

*The number of crossovers in Case 1 is derived directly from the results of each regression. Case 2 uses only those coefficients which are significant at the 10% level.

similarity between these two cases would tend to continue down to the number of factor-intensity crossovers.

As to comparison with Type I, it may be hypothesized that, since Types II and III involve the existence of curvatures in the non-linear "substitution curves" in the VES case, they might produce more cross-over points than Type I. There is however no clear indication in our empirical results to verify this expectation. It was found that even between a pair of non-linear "substitution curves," we rarely observed more than one crossover point in the graphs we plotted. Moreover, since Case 2 allows empirical determination of the form of the production function by utilizing only significant parameters, results of the three regression-types would tend to be more similar than they would otherwise be as in Case 1. This may be the major reason for the apparent lack of significant difference in the number of crossover points among Types I-III in Case 2.

We shall now consider the relative frequency or probability of crossovers. For non-linear "substitution curves," it is difficult to determine a priori the total number of theoretically possible intersections. Among the 14 graphs of "substitution curves" (not reproduced), it has been found to be very rare for two curves to cross more than once in the economically relevant range (i.e. in the positive quadrant). Suppose any two curves can intersect only once (as in the Minhas-Leontief-Philpot analyses), then for 17 industries the theoretically possible number of crossovers is 136 (i.e. C_2^{17}). If we allow that each pair of curves may cross twice, then the number of possible intersections

becomes 272 (i.e. $2xC_2^{17}$). These numbers may be used to approximate theoretical values which enter into the denominator in computing the probability of crossovers.

Using Case 2 results from Type I regressions, the probability is in the vicinity of 45%, which in percentage terms far exceeds Leontief's finding of 17 out of 210 possible crossovers [4]. However, such a probability argument which follows Leontief's is totally arbitrary. It should be realized that the total number of possible crossovers--C_2^{21} in Leontief's case--is purely mathematical, since a certain number of them (which conceptually can be very substantial) may occur in the negative range for which there is no economic relevance, and which should therefore be discarded. Adjusting for these, the "probability" of crossovers would increase.

Using Case 2 results from Types II and III regressions, the "probability" of crossovers is again in the same vicinity of 45% if 136 is considered the total number of possible crossovers, and half of this percentage if 272 is used instead. One might argue that mathematically speaking the theoretical number of possible intersections can be any positive integer times C_2^{17} for non-linear "substitution curves." However, from our experience, we have seen no empirical evidence of each pair of curves intersecting more than twice. And again, allowance must also be made for intersections in the non-relevant (negative) range. Making these adjustments, the actual probability of crossovers is anybody's guess.

It should however be recognized that the absolute number of cross-

overs is of more economic significance than the relative number and is the appropriate figure to reckon with. In this study, we found found the possibility of 48 to 77 crossovers in 17 industries if production functions are ubiquitous but equal relative factor prices are not. This should suggest that the phenomenon of factor reversal may be very important in the theory of international trade.

V Summary and Conclusion

The phenomenon of factor reversal can occur when a combination of production functions, notably members of the VESh family, exists. Previous studies by Minhas and Leontief on the subject utilized the CES function. Philpot used an extension of it, the CESh, which was a step in the right direction in terms of specification-error analysis. Our approach adds two more terms to explain the average productivity of labor, implying a more extended family of production functions.

In enumerating the number of factor-intensity crossovers within the "relevant range," care was taken to distinguish between parameters that are significant and those which are not. It was found that lumping significant with non-significant estimated parameters together tends to distort the number of crossovers. Also, the number of crossover points may be sensitive to the specification of the form of the production function.

The result of our limited study using only U. S. data for 1957 strongly suggests that the existence of factor reversals cannot be ignored. If the production functions of the 17 industries studied are universally applied under varying factor-price conditons, the absolute number of possible factor reversals may be rather common.

This has immediate implication for Leontief-type tests of the Heckscher-Ohlin theorem of international trade. Furthermore, international factor-price non-equalization is a reality which would be more consistent with possibilities of factor reversals than without in the present context of international trade.

REFERENCES

[1] Arrow, K. J., H. B. Chenery, B. S. Minhas, and R. M. Solow, "Capital-Labor Substitution and Economic Efficiency," Review of Economics and Statistics (Aug., 1961), 225-250; reprintedin A. Zellner (ed.), Readings in Economic Statistics and Econometrics (Boston: Little Brown and Co., 1968), 364-508.

[2] Hildebrand, G. H. and T. C. Liu, Manufacturing Production Functions in the United States, 1957 (Ithaca, New York: New York State School of Industrial and Labor Relations, Cornell University, 1965).

[3] Leontief, W. W., "Domestic Production and Foreign Trade: The American Capital Position Re-examined," Proceedings of the American Philosophical Society (Sept., 1953), 332-49.

[4] _____, "An International Comparison of Factor-Costs and Factor Use--A Review Article," American Economic Review (June, 1964), 335-445.

[5] Lu, Y. C. and L. B. Fletcher, "A Generalization of the CES Production Function," Review of Economics and Statistics (Nov., 1968), 449-52.

[6] Minhas, B. S., "The Homohypallagic Production Function, Factor-Intensity Reversals, and the Heckscher-Ohlin Theorem," Journal of Political Economy (April, 1962), 138-156.

[7] _____, An International Comparison of Factor Costs and Factor Use (Amsterdam: North-Holland Publishing Co., 1963).

[8] Philpot, G., "Labor Quality, Returns to Scale and the Elasticity of Factor Substitution," Review of Economics and Statistics (May, 1970), 194-9.

Appendix

A. The Data

The data for the regression analysis were obtained from Appendix I in Manufacturing Production Functions in the U. S., 1957, by Hildebrand and Liu [2].

V: Value added in dollars in 1957.

L: Employment (production workers and non-production employees in man-hours).

L_p: Employment of production workers in man-hours.

w: Average wage and salary rate for production workers and non-production employees in dollars per man-hour

w_p: Average wage rate for production workers in dollars per man-hour.

K_g: Gross book value (owned) of plant and equipment in dollars.

K'_g: Gross book value (owned plus rented) of plant and equipment in dollars.

K_n: Gross book value (K_g) minus accumulated depreciation and depletion.

B. Tests of Hypotheses

The data were used to fit the 14 regressions for each of the 17 industries. The regressions are grouped into 3 basic types. Corresponding to each regression type is a particular form of production function.

Type I (CES form)

1. $\log V/L = \log a + b \log w + u_1$

2. $\log V/L_p = \log a + b \log w_p + u_2$

Type II (VES1 form)

3. $\log V/L = \log a + b \log w + c \log K_g/L + u_3$

4. $\log V/L = \log a + b \log w + c \log K'_g/L + u_4$

5. $\log V/L = \log a + b \log w + c \log K_g/L_p + u_5$

6. $\log V/L_p = \log a + b \log w_p + c \log K_g/L_p + u_6$

7. $\log V/L_p = \log a + b \log w_p + c \log K'_g/L_p + u_7$

8. $\log V/L_p = \log a + b \log w_p + c \log K_n/L_p + u_8$

Type III (VESh form)

9. $\log V/L = \log a + b \log w + c \log K_g/L + d \log L + u_9$

10. $\log V/L = \log a + b \log w + c \log K'_g/L + d \log L + u_{10}$

11. $\log V/L = \log a + b \log w + c \log K_n/L + d \log L + u_{11}$

12. $\log V/L_p = \log a + b \log w_p + c \log K_g/L_p + d \log L_p + u_{12}$

13. $\log V/L_p = \log a + b \log w_p + c \log K'_g/L_p + d \log L_p + u_{13}$

14. $\log V/L_p = \log a + b \log w_p + c \log K_n/L_p + d \log L_p + u_{14}$

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